Published by Institute of Physics Publishing for SISSA
Received: *October 22, 2004*
Revised: *March 7, 2005*
Accepted: *March 18, 2005*
Published: *April 15, 2005*

Nonsingular 4d-flat branes in six-dimensional supergravities

V.P. Nair
Physics Department, City College of the CUNY
New York, NY 10031, U.S.A.
E-mail: vpn@sci.ccny.cuny.edu

S. Randjbar-Daemi
Abdus Salam International Centre for Theoretical Physics
Trieste, Italy
E-mail: seif@ictp.trieste.it

Abstract: We show that six-dimensional supergravity models admit nonsingular solutions in the presence of flat three-brane sources with positive tensions. The models studied in this paper are nonlinear sigma models with the target spaces of the scalar fields being noncompact manifolds. For the particular solutions of the scalar field equations which we consider, only two brane sources are possible which are positioned at those points where the scalar field densities diverge, without creating a divergence in the Ricci scalar or the total energy. These solutions are locally invariant under $\frac{1}{2}$ of $D = 6$ supersymmetries, which, however, do not integrate to global Killing spinors. Other branes can be introduced by hand by allowing for local deficit angles in the transverse space without generating any kind of curvature singularities.

Keywords: Field Theories in Higher Dimensions, p-branes, Supergravity Models, Cosmology of Theories beyond the SM.

JHEP03(2005)049

Contents

1. Introducion

The idea of solving the cosmological constant problem by regarding our universe as a brane in more than four dimensions opens up new interesting possibilities for attacking this fundamental question [1]. In the context of the brane-world models the uniform part of the cosmic energy is indistinguishable from the brane tension. The nonzero tension of a physical three-brane, on which the particles of the standard model are localized, may be compensated by a higher dimensional curvature or a bulk cosmological constant, without generating curvature in the brane.

In a recent work it was shown that in a nonsupersymmetric six-dimensional model of gravity coupled to a sigma model targeted on any Kähler target space one can obtain nonsingular solutions in the presence of flat three-branes with positive tensions [2]. The branes in that paper could be viewed as vortices distributed on a two-dimensional compact manifold with the topology of a two-sphere S^2. The vorticity of a brane can be thought of as an Aharonov-Bohm phase acquired by a scalar field when a complete rotation around the brane is made. There is a single relation between the tensions of the three-branes and their vorticities and it was speculated that the time-variation of the vorticity of each brane may account for the adjustment of the four-dimensional cosmological constant. To obtain such a solution, however, it was necessary to assume that the six-dimensional cosmological constant was zero. It was indeed shown subsequently in [3] that a nonzero six-dimensional cosmological term disturbs the flatness of the three-branes.

In this paper we shall find solutions in six-dimensional supergravity models where a bulk cosmological constant is forbidden by supersymmetry. We shall show that the tensions of the branes can be arbitrary and positive subject to a single linear relation of the type stated in [2] . The solution for the scalar field will be implicit and more involved than the simple mappings given in [4, 2, 3]. Flat three-brane solutions with a singularity at the boundary of the transverse space are known to exist in $D = 6$ supergravities with scalars described by a nonlinear sigma model [4, 5]. This singularity is, unfortunately, located at a finite proper distance from the brane or from any other point on the transverse space. Further, although the Killing spinor equations have local solutions far away from the brane, they do not integrate to a single-valued global solution [4].

The aim of this paper is to present nonsingular solutions in a supergravity model, where like in [4], apart from gravity, the only other active fields will be a set of scalar fields with dynamics governed by a nonlinear sigma model lagrangian targeted on some noncompact hyperbolic or quaternionic manifold. Such scalar fields exist in all supergravity models in six dimensions. Our construction will lead to a smooth transverse space with no boundaries and with an Euler number of +2. Like the solution in [4, 5] our solution in this paper will also be only locally invariant under 1/2 of $D = 6$ supersymmetries. These local supersymmetries, however, do not integrate to globally single valued Killing spinors.

Similar attempts to solve the cosmological constant problem have been made in the past by invoking the magnetic monopole compactifiction in a six dimensional theory of gravity coupled to Maxwell field and in the presence of a six-dimensional cosmological constant [6, 7]. The Kaluza-Klein solution of this system needs a very special tuning of six-dimensional cosmological term versus the radius of the internal S^2 in order to have a flat four-dimensional part [8]. A similar tuning is also necessary in the $(1, 0)$ six-dimensional gauged supergravities of the type constructed in [10, 11], although in this case the $Minkowski_4 \times S^2$ turns out to be the unique maximally symmetric solution which also preserves $\frac{1}{2}$ of the supersymmetries [12]. These theories also have three-brane solutions but with negative tensions and/or singular transverse spaces [12]–[14].[1]

The solution of the scalar field equation in this paper is akin to the one studied in the context of the string comic string in [15]. There, the complex scalar field is the modulus of a T^2, therefore, modular invariance plays a very important role. In our case the restriction imposed by modular invariance is needed for the single valuedness of the space time metric. For this reason our solution for the metric will be somewhat different and, unlike the metric in [15], the singularities in our metric will be smoothed out. This smoothing out necessarily introduces delta function type terms in the Einstein equations whose natural interpretation is the presence of three-branes. Demanding regularity at infinity and positivity of the tensions of the branes will impose more restrictions on our solution to the scalar field equation compared to the solution in [15]. However, unlike the monopole solutions of [6, 7], which require tuning of the parameters to achieve flatness,we shall need a single linear relation on the tensions. The single relationship between the tensions is dictated by the requirement that the geometry becomes spherical away from the branes, as in [2]. This reference also introduced the notion of vorticity for the brane solutions, but, because of the complexity of the scalar field ansatz, it is not clear to us how we can extend this idea to the present case in a straightforward way.

The plan of this paper is as follows: In section 2 we present the solution and show its regularity. In section 3 we examine the conditions for unbroken supersymmetries. Section 4 is dedicated to a brief summary.

[1]Some of these issues will be further discussed in a forthcoming paper by C.Burgess et al.

2. The solutions

Turning to the details of the solutions, consider a $D = 6$ supergravity with scalars in the hyper or tensor multiplets. Such scalars cover a noncompact hyperbolic or quaternioic manifold of the type $\mathrm{SO}(n,1)/\mathrm{SO}(n)$ for the scalars in the tensor multiplet or $\mathrm{Sp}(n,1)/\mathrm{Sp}(n) \times \mathrm{Sp}(1)$ for the scalars in the hypermultiplet. (In this latter case some other quaternionic Kähler manifolds are also possible [16]. Our discussion will be applicable to all of them.) We shall set all of the scalar fields to zero apart from a single complex one which we shall denote by φ. The metric in the two-dimensional subspace of the scalar manifold is given by

$$ds^2 = \frac{d\varphi\, d\bar{\varphi}}{(1 - |\varphi|^2)^2}\,. \tag{2.1}$$

A simple holomorphic change of coordinates $\varphi = (\tau - i)/(\tau + i)$ will then bring it to the form

$$ds^2 = -\frac{d\tau d\bar{\tau}}{(\tau - \bar{\tau})^2}\,. \tag{2.2}$$

We shall look for solutions of the Einstein's equations of the form

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + e^{\phi(z,\bar{z})} dz d\bar{z}\,, \tag{2.3}$$

where $\mu, \nu = 0, 1, 2, 3$ and $(z, \bar{z})$ is a local complex coordinate in the extra two dimensions. In this coordinate system the relevant field equations reduce to

$$\partial\bar{\partial}\tau + 2\,\frac{\partial\tau\bar{\partial}\tau}{\bar{\tau} - \tau} = 0 \tag{2.4}$$

for the scalar field and

$$\partial\bar{\partial}\phi = \frac{\partial\tau\bar{\partial}\bar{\tau}}{(\bar{\tau} - \tau)^2} - \frac{1}{2M^4} \sum T_i \delta^{(2)}(z - z_i) \tag{2.5}$$

for the metric function of the transverse two-space. In the last equation, M is the six-dimensional Planck mass and T_i are the tensions of the brane sources. Note that, unlike in the case of the nonsupersymmettric model, in the present case, the $D = 6$ supersymmetry does not allow for an independent coupling constant for the sigma model. Apart from the brane source terms the form of the field equations are completely fixed by supersymmetry. We also note that the field equations are invariant under an $\mathrm{SL}(2, \mathbb{R})$ group acting on τ by fractional linear transformations, viz.,

$$\tau'(z) = \frac{a\tau(z) + b}{c\tau(z) + d}\,, \tag{2.6}$$

where a, b, c, d are real numbers satisfying $ad - bc = 1$.

We first consider the scalar field equation. Clearly any function of z which is independent of $\bar{z}$ solves that equation. In particular, we take the matter field configuration to be given by

$$J(\tau(z)) = \frac{z - a}{z - b}\,, \tag{2.7}$$

where $J(\tau)$ is the modular function known as the absolute invariant [17]. By solving this for τ in terms of z, we get the matter field configuration. Since J is invariant under modular transformations $\mathrm{SL}(2,\mathbb{Z})$ of τ, the inverse will be multivalued unless we restrict to the fundamental domain F_0 of τ. Writing $\tau = \tau_1 + i\tau_2$, this domain is defined by

$$F_0 = \left\{ \tau \mid \tau_2 > 0,\ |\tau| \geq 1, \quad -\frac{1}{2} \leq \tau_1 < \frac{1}{2}, \quad \tau_1 \leq 0 \text{ for} |\tau| = 1 \right\}. \tag{2.8}$$

In this domain, there is a unique inverse. We will define our solution as the configuration $\tau(z)$ where τ takes values in the fundamental region. Other regions correspond to other solutions which are degenerate with this; classically they can be treated as separate and different solutions. Also, since the equations of motion are invariant under a global $\mathrm{SL}(2,\mathbb{R})$ action, we can obtain new solutions by acting with this group on any given solution.

Note that, as far as the τ field equations are concerned, there is an infinite class of solutions of the type suggested here. In principle, we could consider an implicit equation of the form $J(\tau(z)) = f(z)$, where f is any holomorphic function of z. Here we have taken f to be a simple rational function which approaches unity as z goes to infinity. We shall see later that the requirement of the positivity of the tensions of the branes and regularity of the metric as $|z|$ goes to infinity exclude every other possibility for $f(z)$ apart from the simple one given above.

The ansatz for the metric is given by

$$\begin{aligned} e^{\phi} &= \gamma^2 \left[\tau_2 \eta^2 \bar{\eta}^2 \frac{1}{|z-b|^{\frac{2}{12}}} \right] |z-a|^{-2\alpha} |z-b|^{-2\beta} \times \\ &\times \exp\left[\int d^2z' G(z,z') \frac{\bar{\partial}\partial\tau(z')}{(\bar{\tau}-\tau)(z')} \right], \end{aligned} \tag{2.9}$$

where α, β, γ are constants and the Green function $G(z,z')$ satisfies

$$\bar{\partial}\partial G(z,z') = \delta^{(2)}(z-z'). \tag{2.10}$$

The exponential involving the Green function tends to a constant whenever z approaches a or b, which, as we shall see, will be identified with the positions of the three-branes, and it simply vanishes at every other point. $\eta(\tau)$ is the Dedekind η-function. The first factor $\tau_2\eta^2\bar{\eta}^2|z-b|^{-2/12}$ is needed for modular invariance. Since for $z \to b$ the η function behaves like $(z-b)^{1/24}$, the factor $|z-b|^{-2/12}$ ensures that the metric does not vanish as z approaches b.

Notice that, apart from τ_2 and the exponential factor involving $G(z,z')$, e^{ϕ} is of the form $g(z)\bar{g}(\bar{z})$, where $g(z)$ is holomorphic in z. As a result, in the calculation of $\partial\bar{\partial}\phi$, the function g only contributes δ-functions, which can be identified as the brane contribution to the energy-momentum tensor. Since the factor $\eta/(z-b)^{1/12}$ has no singularity at $z=b$, the contribution due to the brane at $z=b$ comes entirely from the factor $|z-b|^{-2\beta}$. The contribution due to $\tau_2 \exp[\int d^2z' G(z,z')\bar{\partial}\partial\tau(z')/(\bar{\tau}-\tau)(z')]$ will exactly match the field part of the energy-momentum tensor. Also the constant prefactor γ in e^{ϕ}, which measures the size of the transverse space, is undetermined by the field equations and is thus a modulus.

We will now turn to an analysis of the scalar field, metric and curvature at various special points, which will yield constraints on the exponents α and β. But before doing so, let us note that the tensions at these points are simply given by

$$T_a = 2\pi M^4 \alpha \qquad T_b = 2\pi M^4 \beta\,. \tag{2.11}$$

This result follows from the substitution of our ansatz for e^{ϕ} in the Einstein equation and then equating the coefficients of the delta functions at $z = a$ and $z = b$.

We will now require that the transverse geometry in the vicinity of the location of the branes should have no curvature singularities; this will put restrictions on α and β. There are three special points to analyze [17]. These points are essentially characterized by the condition of the vanishing of $J(\tau)$ or $\frac{dJ}{d\tau}$. By analyzing our equations above, it is easy to see that these are the only points where there is the danger of curvature singularities.

1. At $z = a$, J and $\frac{dJ}{d\tau}$ vanish and $\tau = \omega \equiv \exp(2\pi i/3)$. Near this point, we can expand τ as

$$\tau \approx \omega + c\,(z-a)^{\frac{1}{3}}\,, \tag{2.12}$$

where c is a constant. τ_2 and the exponential factor in e^{ϕ} are finite at $z = a$, so that the metric has the behavior

$$e^{\phi} \sim \frac{1}{|z-a|^{2\alpha}}\,. \tag{2.13}$$

The geometry has a conical defect at $z = a$. The energy density for matter fields behaves as

$$\mathcal{E} \approx \frac{|c|^2}{27}\frac{1}{|z-a|^{4/3}}\,. \tag{2.14}$$

The energy density has singularity; this will be matched exactly by a similar singularity in $\partial\bar{\partial}\phi$ so that the Einstein equation will be satisfied. Further, this is an integrable singularity, so that there is no pathology in the total energy. In fact, the curvature is given by

$$\begin{aligned} R &= -e^{-\phi}\left[-\frac{|c|^2}{27}\frac{1}{|z-a|^{4/3}} - \pi\alpha\delta^{(2)}(z-a)\right] \\ &\sim |z-a|^{2\alpha}\left[\frac{|c|^2}{27}\frac{1}{|z-a|^{4/3}} + \pi\alpha\delta^{(2)}(z-a)\right] \\ &= e^{-\phi}\left[\mathcal{E} + \pi\alpha\delta^{(2)}(z-a)\right]. \end{aligned} \tag{2.15}$$

We notice that there is a singularity in the curvature if we do not have the term $|z-a|^{2\alpha}$. With this term, we can avoid the singularity by taking

$$\alpha \geq \frac{2}{3}\,. \tag{2.16}$$

In view of the aforementioned relationship between the tension of the brane at $z = a$ and the parameter α we thus obtain a lower bound on T_a, namely

$$T_a \geq \frac{4\pi}{3}M^4\,. \tag{2.17}$$

JHEP03(2005)049

The behavior of τ near $z = a$ requires another comment. Evidently, $(z-a)^{\frac{1}{3}}$ is not single-valued as we go around the point $z = a$; also the value of τ after such a rotation, namely, $\tau(\lambda e^{2\pi i})$, $\lambda = z - a$, is not within the fundamental modular region. However up to first order in λ, there is a modular transformation given by

$$\tau(\lambda e^{2\pi i}) = \frac{\tau(\lambda)+1}{-\tau(\lambda)} \tag{2.18}$$

which maps τ back into the fundamental region. Since the fields are defined up to such a modular transformation, this does not change the physical fields. The factor $\tau_2\eta^2\bar{\eta}^2|z-b|^{-\frac{2}{12}}$ in (2.9) is modular invariant, making the metric invariant, as in [15].

2. As $|z| \to \infty$, $J \to 1$, and $\tau \to i$. Thus asymptotically, we can expand $\tau(z)$ as

$$\tau \approx i + c_2\, z^{-1/2}\,. \tag{2.19}$$

It is then easily seen that the metric behaves as

$$e^{\phi} \sim |z|^{-2\alpha-2\beta-2/12}\,. \tag{2.20}$$

In order for this not to lead to deficit angles at infinity, we need e^{ϕ} to behave as $|z|^{-4}$. This gives a constraint

$$\alpha + \beta + \frac{1}{12} = 2 \tag{2.21}$$

which translates into a constraint among the tensions, viz,

$$T_a + T_b = \frac{11}{6}\, 2\pi M^4\,, \tag{2.22}$$

an identity similar to the constraint derived in [2]. Note that this behavior as $|z|$ goes to infinity also insures that the Euler number of our transverse space is $+2$.

3. As $z \to b$, $J \to \infty$. In this case, the metric has the form

$$e^{\phi} \sim \left[-\frac{1}{2\pi}\log|z-b|\right]\ |z-b|^{-2\beta}\,. \tag{2.23}$$

Without the extra term $|z-b|^{-2\beta}$, there is a curvature singularity, since the Ricci scalar is given by

$$R \approx |z-b|^{2\beta}\frac{1}{(-\log|z-b|)}\left[\frac{1}{4}\frac{1}{|z-b|^2\,(\log|z-b|)^2} + \pi\beta\delta^{(2)}(z-b)\right]. \tag{2.24}$$

Even though the Einstein equation can be satisfied, balancing the singular term of $\mathcal{E}$ with the similar term in $e^{\phi}R$, the curvature itself will be singular unless we choose

$$\beta > 1\,. \tag{2.25}$$

In terms of the tension T_b this implies

$$T_b > 2\pi M^4\,. \tag{2.26}$$

JHEP03(2005)049

Thus, we see that, to obtain regular geometry not only the tensions should be bounded from below by positive quantities, their sum should also be bounded by $(11/6) \times 2\pi M^4$. It can be verified that are no other points at which the curvature behaves badly. The three constraints do have a set of solutions for α and β.

Let us now turn to a possible generalization of equation (2.7) to

$$J(\tau(z)) = \prod_{k=1}^{N} \frac{z - a_k}{z - b_k}\,. \tag{2.27}$$

In order to avoid singularities whenever z approaches any one of the a_k's or b_k's we need to insert a factor $\prod_k |z - a_k|^{-2\alpha_k} |z - b_k|^{-2\beta_k}$ in the ansatz for the metric and restrict each α_k and β_k by

$$\alpha_k \geq \frac{2}{3}\,, \qquad \beta_k \geq 1\,. \tag{2.28}$$

The requirement that e^{ϕ} should behave like $|z|^{-4}$ whenever $|z|$ goes to infinity will then restrict these parameters by the linear relation

$$\sum_1^N (\alpha_k + \beta_k) + \frac{1}{12} = 2\,. \tag{2.29}$$

Clearly we can satisfy all these constraints only for $N = 1$. Presumably the introduction of vorticity as in [2] would allow us more freedom. However, at this stage it is not clear to us how to do this a simple physical way.

The only way that we can keep $\tau(z)$ as a solution of the simple equation $J(\tau(z)) = (z-a)/(z-b)$ and yet introduce more than two branes is to modify the metric by inserting a factor like $|z-a|^{-2\alpha}|z-b|^{-2\beta} \prod_k |z - a'_k|^{-2\alpha_k} |z - b'_k|^{-2\beta_k}$, where $a'_k s$ and $b'_k s$ are different from a and b. In this case, by requiring that α'_k's and β'_k's are smaller than unity, we can have a solution to all the constraints.

3. Supersymmetry

Now we turn to the question of unbroken supersymmetries. Having set all the fields to zero apart from gravity and a pair of real scalars which we have assembled into a single complex field τ, the only Killing spinor equations we need to examine are those of gravitino and the scalars. Although our argument is valid for all (ungauged) supergravities in $D = 6$, to be concrete, in this section, we shall consider hypermatter scalars in ungauged (1,0) supergravities. Such models are chiral and can be obtained from the compactification of the heterotic string on $K3$ [18] or the M theory on $K3 \times S^1/Z_2$ [19]. They parameterize a quaternionic manifold of the form $G/H \times \mathrm{SU}(2)$.

The gravitino as well as the hyperino carry spinor indices of SU(2). Their supersymmetry variation in our bckground is given by [2]

$$\delta\psi_m^r = \partial_m \epsilon^r + \frac{1}{2}\omega_m \Gamma_{45} \epsilon^r + \partial_m \phi^\alpha Q_\alpha^{rs} \epsilon_s \tag{3.1}$$

$$\delta\psi^{\dot r r} = V_\alpha^{\dot r r} \partial_m \phi^\alpha \Gamma^m \epsilon_r\,. \tag{3.2}$$

[2] Here we closely follow the notations of [16] and [20].

In these equations the spinor indices $r, s \ldots$ take values 1 and 2, the index m is tangent to the transverse space and takes the values 4 and 5. ϕ^α denote the two nonvanishing hyperscalars. ω_m and Q in the gravitino equation denote the connections in the transverse space and the hyperscalar manifold respectively. Finally $V_\alpha^{\dot{r}r} = V_\alpha^1 (i\sigma^3)^{\dot{r}r} + V_\alpha^2 (i\sigma^1)^{\dot{r}r}$. We have denoted the components of the zweibein in the tangent space of the scalar manifold by V_α^1 and V_α^2. All we need to know about them is that their only non vanishing components are $V_1^1 = V_2^2$. Finally we quote, without giving the details, that for our background the spin connections ω_m and Q are given by

$$\omega_z = \frac{1}{2}\frac{\partial_z \tau_1}{\tau_2} + \partial_\tau \ln \eta \partial_z \tau - \frac{\alpha}{2}\frac{1}{z-a} - \frac{\beta + 1/12}{2}\frac{1}{z-b} \tag{3.3}$$

$$\partial_z \phi^\alpha Q_\alpha^{rs} = -\frac{i}{4}\frac{\partial_z \tau_1}{\tau_2}(\sigma_2)^{rs} . \tag{3.4}$$

The condition $\delta\psi^{\dot{r}r} = 0$, can be solved provided we restrict the supersymmetry parameter ϵ by

$$\left[\Gamma_4 (\sigma_3)^{\dot{r}r} + \Gamma_5 (\sigma_1)^{\dot{r}r}\right] \epsilon_r = 0 .$$

This condition cuts the number of components of ϵ by half. Upon imposing this condition the gravitino equation $\delta\psi_m^r = 0$ then reduces to a first order linear differential equation for ϵ as a function of z and $\bar{z}$ which in general can be integrated. The solution will, however, be a mutivalued function of z. A full rotation around the branes will not bring ϵ back to its original value. It has been suggested in [4] that by introducing a flat U(1) connection one can eliminate the multivaluedness by an Aharonov-Bohm phase. It will be interesting to find a supergravity model in six dimensions where this idea can be implemented without disturbing other nice features of our solutions.

4. Conclusion

In summary, we constructed nonsingular flat brane solutions in ungauged six dimensional supergravities with a transverse space of Euler number +2. Since our transverse space has no singularities or boundaries the value +2 for its Euler number implies that, topologically, it is a S^2. Thus its volume will be finite, although it is not easy to evaluate it explicitly. The size of the space depends on an integration constant which is not determined at the classical level. It is thus a modulus. It will be interesting to see if temperature effects in a cosmological context can stabilize the value of this radius. This is known to happen in Kaluza Klein cosmology in the absence of branes [21]. Aspects of brane cosmology and the question of self tuning of cosmological constant in co-dimension two have been studied in [22].

Like the solutions in [4] the solutions presented in this paper also break 1/2 of the supersymmetries. The local solution to the Killing spinor equation cannot, however, be integrated to a single-valued global solution. It is suggested in [4] that by coupling the gravitino to a U(1) gauge field and allowing for multivalued Killing spinors one may recover single-valuedness. Such U(1) couplings do indeed exist in the gauged six-dimensional

JHEP03(2005)049

supergravities. However, one difficulty, among others, in implementing this interesting idea is that in this case the scalar fields will have a nontrivial potential and their field equations cannot be solved by holomorphic configurations.[3]

Our construction cannot be considered a completely satisfactory solution to the cosmological constant problem, principally, due to the linear relation among the tensions that we derived in section 3. The introduction of vorticity along the lines of [2], may improve our construction, although we do not have a simple physical way of implementing this idea in the present context.

Acknowledgments

We would like to thank Bobby Acharya for reminding us of the reference [15] The work of V.P.N. was supported in part by the U.S. National Science Foundation grant number PHY-0244873.

References

[1] V.A. Rubakov and M.E. Shaposhnikov, *Extra space-time dimensions: towards a solution to the cosmological constant problem*, *Phys. Lett.* **B 125** (1983) 139;
M. Visser, *An exotic class of Kaluza-Klein models*, *Phys. Lett.* **B 159** (1985) 22 [hep-th/9910093];
S. Randjbar-Daemi and C. Wetterich, *Kaluza-Klein solutions with noncompact internal spaces*, *Phys. Lett.* **B 166** (1986) 65;
G.W. Gibbons and D.L. Wiltshire, *Space-time as a membrane in higher dimensions*, *Nucl. Phys.* **B 287** (1987) 717 [hep-th/0109093];
R. Sundrum, *Compactification for a three-brane universe*, *Phys. Rev.* **D 59** (1999) 085010 [hep-ph/9807348];
N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, *A small cosmological constant from a large extra dimension*, *Phys. Lett.* **B 480** (2000) 193 [hep-th/0001197];
S. Kachru, M.B. Schulz and E. Silverstein, *Self-tuning flat domain walls in 5D gravity and string theory*, *Phys. Rev.* **D 62** (2000) 045021 [hep-th/0001206];
G.R. Dvali and G. Gabadadze, *Gravity on a brane in infinite-volume extra space*, *Phys. Rev.* **D 63** (2001) 065007 [hep-th/0008054];
J.E. Kim, B. Kyae and H.M. Lee, *A model for self-tuning the cosmological constant*, *Phys. Rev. Lett.* **86** (2001) 4223 [hep-th/0011118]; *Self-tuning solution of the cosmological constant problem with antisymmetric tensor field*, *Nucl. Phys.* **B 613** (2001) 306 [hep-th/0101027];
C. Deffayet, G.R. Dvali and G. Gabadadze, *Accelerated universe from gravity leaking to extra dimensions*, *Phys. Rev.* **D 65** (2002) 044023 [astro-ph/0105068];
G. Dvali, G. Gabadadze and M. Shifman, *Diluting cosmological constant in infinite volume extra dimensions*, *Phys. Rev.* **D 67** (2003) 044020 [hep-th/0202174];
U. Ellwanger, *Blown-up p-branes and the cosmological constant*, *JCAP* **11** (2003) 013 [hep-th/0304057].

[2] S. Randjbar-Daemi and V.A. Rubakov, *4D-flat compactifications with brane vorticities*, *JHEP* **10** (2004) 054 [hep-th/0407176].

[3] For an explicit form of the potential in (1,0) supergravity in six dimensions see [20].

[3] H.M. Lee and A. Papazoglou, *Brane solutions of a spherical sigma model in six dimensions*, *Nucl. Phys.* **B 705** (2005) 152 [hep-th/0407208].

[4] A. Kehagias, *A conical tear drop as a vacuum-energy drain for the solution of the cosmological constant problem*, *Phys. Lett.* **B 600** (2004) 133 [hep-th/0406025].

[5] G. Dvali, S. Randjbar-Daemi and Ergin Segin unpublished.

[6] S.M. Carroll and M.M. Guica, *Sidestepping the cosmological constant with football-shaped extra dimensions*, hep-th/0302067.

[7] I. Navarro, *Codimension two compactifications and the cosmological constant problem*, *JCAP* **09** (2003) 004 [hep-th/0302129].

[8] S. Randjbar-Daemi, A. Salam and J. Strathdee, *Spontaneous compactification in six-dimensional Einstein-Maxwell theory*, *Nucl. Phys.* **B 214** (1983) 491.

[9] I. Navarro, *Spheres, deficit angles and the cosmological constant*, *Class. and Quant. Grav.* **20** (2003) 3603 [hep-th/0305014].

[10] A. Salam and E. Sezgin, *Chiral compactification on Minkowski S^2 of $N = 2$ Einstein-Maxwell supergravity in six-dimensions*, *Phys. Lett.* **B 147** (1984) 47.

[11] S. Randjbar-Daemi, A. Salam, E. Sezgin and J. Strathdee, *An anomaly free model in six-dimensions*, *Phys. Lett.* **B 151** (1985) 351.

[12] G.W. Gibbons, R. Guven and C.N. Pope, *3-branes and uniqueness of the Salam-sezgin vacuum*, *Phys. Lett.* **B 595** (2004) 498 [hep-th/0307238].

[13] Y. Aghababaie, C.P. Burgess, S.L. Parameswaran and F. Quevedo, *Towards a naturally small cosmological constant from branes in 6d supergravity*, *Nucl. Phys.* **B 680** (2004) 389 [hep-th/0304256].

[14] Y. Aghababaie *et al.*, *Warped brane worlds in six dimensional supergravity*, *JHEP* **09** (2003) 037 [hep-th/0308064].

[15] B.R. Greene, A.D. Shapere, C. Vafa and S.-T. Yau, *Stringy cosmic strings and noncompact Calabi-Yau manifolds*, *Nucl. Phys.* **B 337** (1990) 1.

[16] H. Nishino and E. Sezgin, *The complete $N = 2$, $D = 6$ supergravity with matter and Yang-Mills couplings*, *Nucl. Phys.* **B 278** (1986) 353.

[17] E.T. Wittakker and G.N. Watson, *A course of modern analysis*, Cambridge University Press 1962;
K. Chandrasekharan, *Elliptic functions*, Springer-Verlag , 1985.

[18] M.B. Green, J.H. Schwarz and P.C. West, *Anomaly free chiral theories in six-dimensions*, *Nucl. Phys.* **B 254** (1985) 327.

[19] A. Sen, *M-theory on $(K3 \times S^1)/\mathbb{Z}_2$*, *Phys. Rev.* **D 53** (1996) 6725 [hep-th/9602010].

[20] S. Randjbar-Daemi and E. Sezgin, *Scalar potential and dyonic strings in 6D gauged supergravity*, *Nucl. Phys.* **B 692** (2004) 346 [hep-th/0402217].

[21] S. Randjbar-Daemi, A. Salam and J. Strathdee, *On Kaluza-Klein cosmology*, *Phys. Lett.* **B 135** (1984) 388;
T. Bringmann and M. Eriksson, *Can homogeneous extra dimensions be stabilized during matter domination?*, *JCAP* **10** (2003) 006 [astro-ph/0308498];
T. Bringmann, M. Eriksson and M. Gustafsson, *Cosmological evolution of universal extra dimensions*, *Phys. Rev.* **D 68** (2003) 063516 [astro-ph/0303497].

JHEP03(2005)049

[22] J.M. Cline, J. Descheneau, M. Giovannini and J. Vinet, *Cosmology of codimension-two braneworlds*, *JHEP* **06** (2003) 048 [hep-th/0304147];
J. Vinet and J.M. Cline, *Can codimension-two branes solve the cosmological constant problem?*, *Phys. Rev.* **D 70** (2004) 083514 [hep-th/0406141].